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Measuring Optical Transfer Functions of Lenses with the Aid of a Digital Computer

Abstract: The problems involved in lens testing, as opposed to the testing of one lens under one set of conditions, are of sufficient magnitude and complexity that the use of a computer is almost imperative to make the job practical. A machine and method for evaluation of lenses is described which employs a digital computer as a major component, has no restrictions as to the quality of lenses which may be accomodated, and requires no precision targets or auxiliary optics of any type. The results are in a form directly applicable to predictions of performance of complicated systems where the lens is one of several linear dissipators.

The reader is introduced to lens testing considerations and a brief discussion of methods, followed by a detailed description of a specific implementation and computerized approach. Next, the basic mathematics involved, including a practical procedure for truncating a Fourier series, are explained in some detail. Finally, examples of measured output and machine accuracy and stability examination are given.

Introduction

The purpose of testing a lens is to permit a user to predict its performance under some specified conditions—for example, to predict the energy distribution in the image of a given object. There are two types of ultimate use for lenses for which predictions of performance are desirable. Examples of these are ordinary pictorial photography and applications wherein the lens is a component of a system in which signals are transferred. While these are basically the same in their purely physical aspects, the first includes a subjective element for which a theoretical guide is basically lacking—hence it is specifically ruled out of consideration. We confine ourselves then to the evaluation of lenses which are used as components of purely physical systems.

There are at least three major areas through which the ultimate lens use dictates the form and quantity of the data which must be taken and reduced, and the method of obtaining such data. These areas involve the characteristics of image formation, the wide ranges of resolution which may be built into lenses in general, and the accuracy and reliability of the measurement methods, as determined by the end use of the measurements.

1. There is general agreement that the optical transfer function (OTF) is a suitable merit function for a lens. Let it be clearly understood that no single measurement, OTF or otherwise, is sufficient to characterize a lens. To have any practical meaning an OTF must be accompanied by a set of specifications indicating the conditions under which it was measured, including wavelength of the light radiated from the object, conjugate distances, field angle, degree of coherence, aperture, defect of focus, azimuth angle, etc. Each combination of these may yield a significantly different OTF. With several steps of each parameter, the number of possible combinations can easily be several hundred.[1]

The OTF is merely a Fourier interpolation function which, when inverted, gives the shape of the physical energy distribution in the image of a line source. Regardless of the theory of how the phenomenon comes about, the image of a point in the object field is a distribution of energy in the image space—the shape and size of this distribution is by no means an invariant property of a particular lens under study. A close examination of an average photograph will show that the quality falls off in the corners of the frame or with lack of proper focus, and may vary with the color of light, as noted above. The shape and size of the dissipation function clearly is not invariant, and this is a major part of the lens testing problem. These observations being true, given a lens and the command "Test it," one can measure an almost infinite number of OTF's. The ultimate use of the lens is an inseparable part of the test, serving to define a restricted set of consitions which, in turn, define a number of OTF's to be measured.

2. The second area involves the wide resolution range for which lenses are designed. Two extremes from the

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experience of our laboratory will illustrate this point. One case is represented by a lens for an optical character reader, deliberately designed to cut off at 20 cycles/millimeter over a wide field and great depth of focus in order to filter against paper noise. At the other extreme was an example of the lens designer's art used in the photoengraving production of small parts. In the first case, the emphasis was on a wide range at the red end of the spectrum, a great depth of focus, and a deliberate resolution restriction. In the other case, the main concern was the highest possible resolution, with critical focus control and monochromatic light assumed. The testing method must be able to accomodate this broad range of requirements.

3. The third area embraces the fact that measurements without an indication of accuracy and reliability are worthless in an engineering context. The measuring machine and data reduction method must be capable of evaluating and exhibiting their own errors, and preferably must be adjustable to accomodate these continuously to the final use. This matter will be treated in some detail in a subsequent section.

Practical methods

The various methods which have been either implemented or proposed for the measurement of OTF fall into two general categories—those requiring interferometry and those utilizing moving targets. Interferometric measurement in general is limited to monochromatic light. The moving target approach demands precise manufacture and evaluation of the targets to be used as measurement standards. All methods claiming utility as engineering tools should give results which are easily combined with other data, implying interpolation and thus extensive computation. This is a major reason for the existence of the OTF.

A serious limitation appears in those measurement implementations which employ auxiliary optical elements in the test path. The accuracy of the final result is, of course, defined in part by the accuracy with which these elements are characterized. The upper limit of resolution of the entire system is dictated by the quality of these same elements. To accurately evaluate lenses of the highest quality, such as commercial microscope objectives, all auxiliary elements must be eliminated.

It is not the intent of this paper to review all methods of lens evaluation which may be considered. The interested reader may find them in the fairly voluminous literature.[2]

As far as is known, these devices have been strictly analog machines, yielding an output on a paper chart, oscilloscope, photographic plate, or meter reading. These output forms are distinctly disadvantageous. When one wishes to employ the computational power of the OTF for general system performance prediction, these data must be reduced to computer readable form. In addition, the

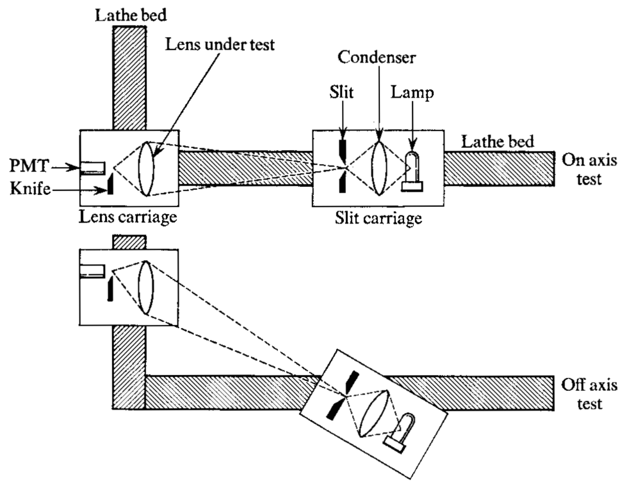
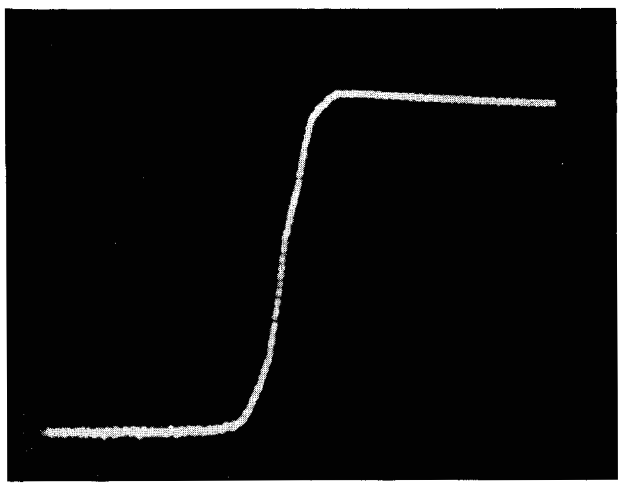


Figure 1 Measuring system schematic.

Figure 2 Typical edge function—6″ f/2.8 lens, on axis at 3:1 magnification, using Wratten #55 filter.



extraction of the phase information to a degree of accuracy approaching that of the modulation measurement is extremely difficult by strictly analog or mechanical means, and impossible in some cases.

When one considers the fact that all of the information necessary to obtain both the modulus and phase portions of an OTF is contained in the edge function, it is apparent that the recording system may be simple in principle, requiring only the minimum of analog instrumentation, by placing the burden of the necessary analysis on a high-speed digital computer. Significant problems in metrology must be considered, but these are inherent in all photoelectric measurements and are immediately exposed, rather than being disguised in the instrumentation. The desired information may be extracted through properly controlled computational processes to the limit of the measurement accuracy. We shall now describe a physical implementation which has been specifically engineered to encompass as much of the general lens testing problem as is practical.

Edge function implementation

The essentials of the measuring technique are shown in Fig. 1. An incoherently illuminated slit of known small dimensions is imaged by the lens under test in the neighborhood of a knife edge and is moved in a direction normal to the edge. As the energy distribution moves past the edge, the radiant flux is collected by a photomultiplier as a function of the position of the energy distribution relative to the edge. The resultant waveform, shown for a typical case in Fig. 2, is the edge function $E(x)$ for the lens under one set of conditions. In practice, $E(x)$ is defined by discretely measured points on this function.

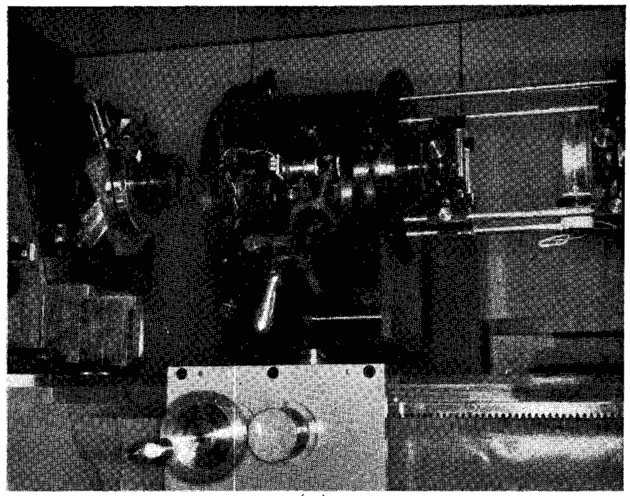
Either the slit or the knife may be moved to accomplish the above process. Mechanical considerations dictate that the critical motion be on the long conjugate of the lens.

Figures 3a and 3b are views of the mechanical parts of the machine, made in two sections and assembled on rigid ways at right angles. In the center of Fig. 3a is shown the illuminator and the slit housing. The illuminator is a ribbon-filament tunsten lamp, imaged by a high-aperture condenser into the plane of the moveable slit. The high aperture of the condenser is necessary to insure filling of the aperture of the lens under examination. Filters of any desired bandwidth within the range of radiation of the tungsten source may be inserted between the condenser lens and the slit to permit measurement under various chromatic conditions.

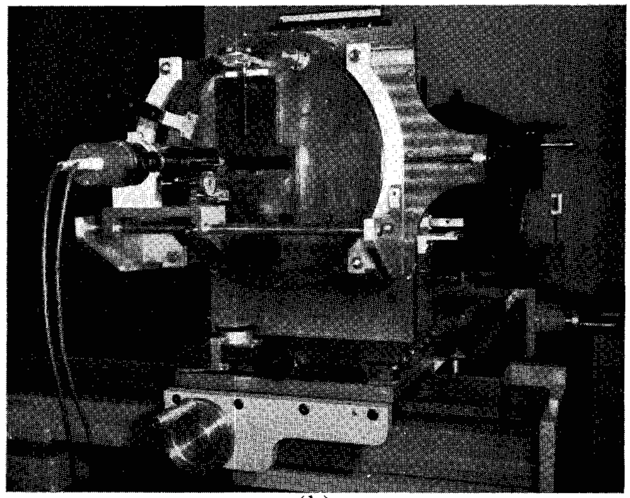
The slit is moved across the optical axis by a flexure pivot torque motor. Calibration of the slit displacement independent of the flexure motor drive current is furnished by a 22-cycle/millimeter ruling, moving with the slit in an auxiliary optical path formed by a microscope objective, lamp, and photodiode. The entire slit mounting and illuminator assembly may be pivoted around a vertical axis through the slit to accomodate off-axis measurements, and also is capable of rotation about the optical axis to permit sweeps at various azimuths.

As shown in Fig. 3a, the slit housing may be tilted out of the mounting to remove the slit from the optical axis, allowing the operator to insure that the aperture of the of the lens under examination is filled.

Figure 3b shows the assembly which supports the lens under examination, the knife edge, photomultiplier, and viewing microscope. The knife edge is adjustable in rotation to bring it perpendicular to the motion of the slit. The photomultiplier and viewing microscope are mounted on a



(a)



(b)

Figure 3(a) (left to right) Lens under test in lens mount, slit housing, condenser and illuminator assemblies with cover removed.
(b) Knife edge, photomultiplier, and viewing microscope. Slit housing and illuminator in background, right.

bar slide so that either may be placed directly behind the image of the slit formed by the lens. The entire lens and knife edge mounting assembly may be moved to the left for off-axis measurements.

Because of the lack of auxiliary optics in the test path, the upper limit of lens quality which may be accommodated is determined primarily by the quality of the knife edge and the slit, and the parallelism between them. This limit can conceivably be in excess of 1000 cycles/millimeter.

The characteristics of the controls in the present machine were dictated by the measurements that must be performed to obtain data from an edge trace. Past experience with earlier versions indicated that extensive control over the

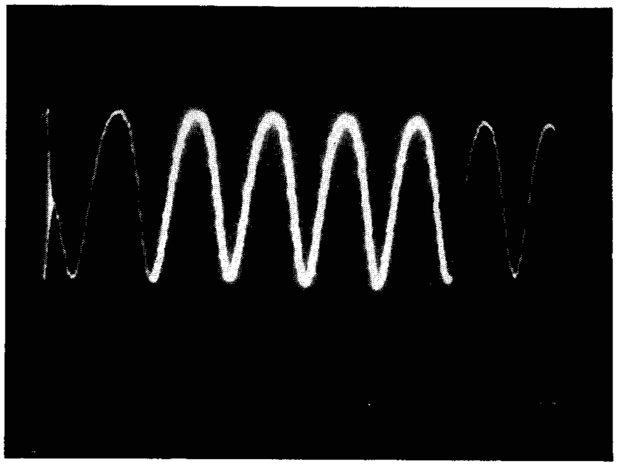


Figure 4 Calibration ruling waveform, with 4 complete cycles intensified.

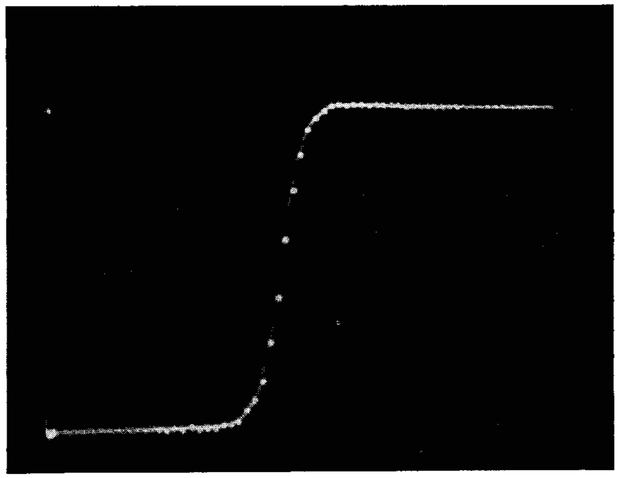


Figure 5 Magnification measurement slits separated by 260 microns. 6 in. f/4 lens, on axis at 1.25:1 magnification, Wratten #55 filter.

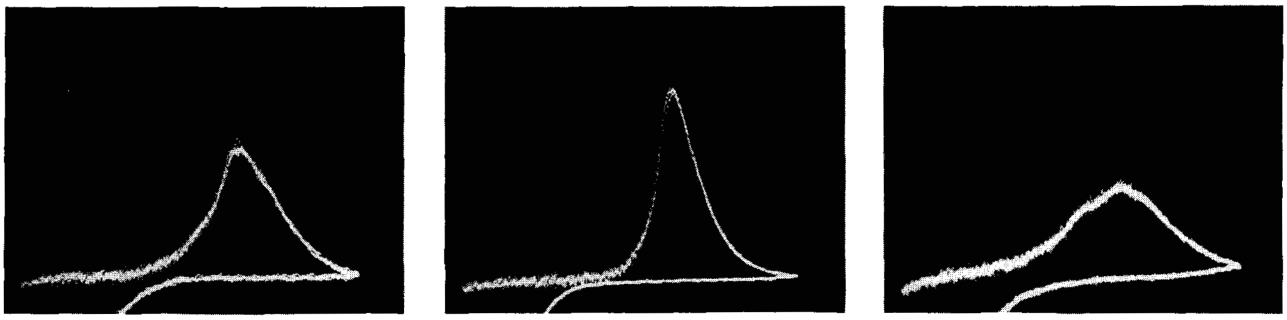
range of slit movement and the density and number of data points is required for maximum ease and utility. For calibration purposes, it is necessary to determine the slit displacement in the image plane between the first and last recorded data points. This implies an ability to measure the object plane slit displacement, thus finding the image displacement by a priori knowledge of the system magnification, or some method of measurement in the image plane. In the cases where measurement of the lens magnification is desired, a means of relating distances in both image and object planes is necessary.

These requirements were met with a digital drive system for the flexure pivot motor. A 1000-level digital/analog converter drives the flexure motor and a monitor oscilloscope in parallel. The x-displacement of the monitor oscilloscope is thus synchronized to the slit and ruling motions. Digital control and clocking circuits, combined with a series of manual switch registers, allow the operator to intensify the trace of the monitor oscilloscope at points representing any of the 1000 addresses of the flexure motor, or over areas bounded by known addresses. By displaying the waveform of the photodiode in the auxiliary optical path, the relationship between the ruling and the flexure motor addresses may be determined. A typical measurement of this type is shown in Fig. 4, where four complete cycles of the calibration ruling waveform have been intensified by the control circuits. The number of flexure motor addresses represented by this intensified region thus establishes the width of the fundamental measurement interval in the object plane.

The fundamental interval may be measured in the image plane by inserting therein parallel slits of known separation. If the image of the object slit is swept across the parallel slits, and the output waveform from the photomultiplier is displayed on the monitor oscilloscope, the above procedure leads to the establishment of the fundamental interval width in the image plane. The ratio of the object and image widths is, of course, the system magnification. This procedure may also be applied to set the system magnification at some desired value for a series of tests. A typical magnification measurement is shown in Fig. 5. Slit separation in this example is 260 microns.

With the knife edge in the image plane, and the object slit in motion, the photomultiplier waveform is the edge trace of the lens. The operator may adjust the machine for focus and centering of the trace within the sweep range of the flexure motor. Care must be taken to insure that the sweep of the image is from minimum to maximum intensity, encompassing all of the observable flare which may be present in the lens. Several ranges of sweep width are available to the operator to insure that this condition is met.

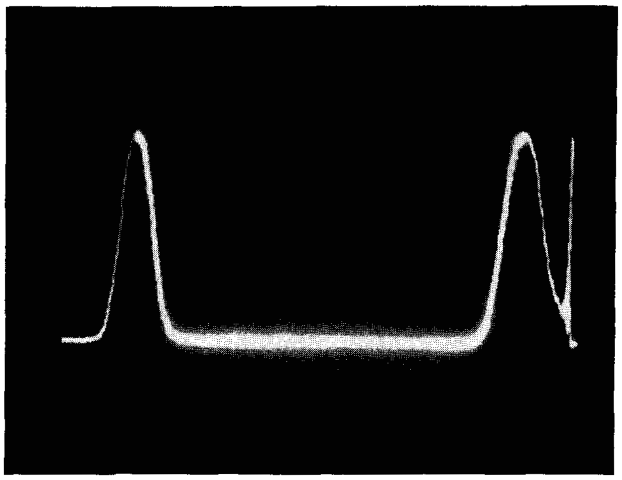
While an approximate focal setting for the lens under test may be found by visual inspection of the image, this depends on a subjective judgement by the operator and is not sufficiently accurate for the final measurement. Critical focal adjustment is done electronically, by peaking an approximate derivative of the edge trace. This adjustment is very sensitive and largely independent of the operator, yielding excellent repeatability both in mechanical positioning and OTF measurements. The derivative waveforms at and near focus for a typical case are shown in Figs. 6(a), 6(b), and 6(c). The image is swept rapidly back and forth across the knife edge, so that a family of derivative



(a) (b) (c)

Figure 6 Derivative waveforms. Same lens as in Fig. 5 (a) Knife edge positioned 0.030″ behind best focal position (b) Knife edge at best focal position (c) Knife edge positioned 0.030″ ahead of best focal position.

Figure 7 Edge function of Fig. 2 with intensified data points shown.



curves is generated on the monitor oscilloscope screen, allowing the operator to ascertain the immediate result of his adjustments.

The system magnification, sweep range, and focus having been established, the operator must determine that a sufficient number of data points will be recorded from the edge trace. The digital control circuits are used to intensify the trace at each address where a data point will be taken. The number of fundamental intervals between the data points is under control of the operator, as is the beginning and end of the data gathering area within the entire sweep range. An edge function with displayed data points is shown in Fig. 7.

The final operation prior to the data collection is to insure the reliability of the photoelectric measurement. The photomultiplier load and the machine sample time are simultaneously selected to insure the accuracy and reliability dictated by the ultimate use of the measurements, and are determined by the statistics of random photon arrival.[3] The minimum sample time which may be employed is determined by seismic vibrations in the building and is set to minimize these contributions.

All remaining operations proceed under computer control. The object slit is moved sequentially over intervals of equal length, and the photocurrent representing the total flux passing the knife edge at each point is digitized. The flexure motor address at each data point, and the photomultiplier current readings, are transferred directly to the computer to become the x and $E(x)$ values of the edge function.

Computer interface

A modified IBM 1620-1 computer serves as the data acquisition and reduction system. Peripheral units, one of which is the lens tester, communicate with the central processing unit by simulating either a paper tape punch or paper tape reader, through an external control system attached to the paper tape channel. The lens tester is one of several selectable input devices, supplying x and $E(x)$ values directly into core memory through the use of the standard computer instruction set.

To facilitate off-line use of the tester for non-computational measurements, such as field curvature, and to allow data acquisition via punched cards when the computer is not immediately available, the basic controls were implemented in the machine hardware, rather than being incorporated into the computer software. This approach provides an efficient balance between computer software, external hardware, and operator control.

Computer program

A conversational program written in conventional FORTRAN provides flexibility and operational ease, both in the setup phase and the data reduction operations. The control system previously described provides certain constants which the operator enters at the console typewriter. For instance, entry of the number of calibration ruling cycles intensified over a certain number of intervals causes calculation of the fundamental interval width in the object plane. The system magnification and the number of fundamental intervals in the image between the parallel slits are directly related. Specifying either parameter will thus cause calculation and display of the other. All parameters calculated or entered during the setup phase are retained in the computer memory for use in the data reduction. The operator is free to repeat the adjustments of the machine until the desired measurement conditions have been established, with the burden of the necessary arithmetic being assumed by the computer.

The program allows the operator to specify parameters for the calculations and the quantity and form of the output data, prior to the data taking. For example, he may enter at the typewriter demands for any or all of the following: direct measurement of the edge function, or entry from a previously punched card deck; tabulations of the edge function, line spread function, and the amplitude and phase of the OTF at arbitrary intervals; visual display and graphic plots of these functions; and a punched card deck containing the edge function data and essential parameters for possible future use. The computer fills these requests automatically. When the metrological job is completed, and data reduction begins, the operator is free to set up the next test while the computer finishes the calculations. However, the operator may intervene at any point to alter the course of the computer operation.

It should be noted that the burden of the mathematical analysis has been relegated to the computer, both for the setup operations and the data reduction. Calculations involved in the edge function approach, properly implemented, are of such magnitude that execution by any other means is a practical impossibility. Employment of the computer in this manner reduces the complexity required in the metrological unit, while simultaneously providing extreme flexibility and computational accuracy.

The flexibility available to the operator by virtue of the computer is of greatest value when the desired test conditions for a given lens are only partially defined. In this case, preliminary tests may dictate the quantity and nature of subsequent tests, with detailed analysis of any one or all tests possible at will until the desire result is obtained. In addition, the computer program maintains a continuing check on the measurement and reduction process to extract all of the information from the data, within limits set by the machine or the ultimate use of the measurements, and to report the magnitude of any errors that may be present. Finally, the computer performs a small amount of operator error checking. For example, a request for the calculation of the OTF at a frequency in excess of the highest reliable frequency is indicated by an appropriate error message and the setting to zero of all requested amplitudes at and beyond that frequency. Also, the presence of extreme system or measurement errors may be easily noted.

Mathematical considerations

A point in the object plane of a lens is represented in the image plane by a dissipation function $P(x, y)$. The line spread function $L(x)$ is defined as

$$L(x) = \int_{-\infty}^{\infty} P(x, y)\, dy.$$

The OTF is the Fourier transform of $L(x)$.

The edge function $E(x)$ is defined as

$$E(x) = \int_{-\infty}^{x} \int_{-\infty}^{\infty} P(x, y)\, dy\, dx = \int_{-\infty}^{x} L(x)\, dx$$

and hence $L(x) = d/dx\, E(x)$. These relationships are well known in the literature, notably R. V. Shack[4] and E. H. Linfoote.[5]

The first job that the computer must perform is to "fit" the discrete data points gathered in the metrological operation to an analytical function $E(x)$. Since the final result is to be a Fourier transform, it is natural to choose a trigonometric polynomial to represent the edge function data points.

$$y = mx + b + \sum_{k=1}^{\mathrm{MAX}} B(k) \sin \pi kx/L.$$

In this representation of a function initially described by discrete points, the critical link between the geometrical model and the real world of physical measurements is the rule which determines the truncation of the approximating series. Consider only

$$H(x) = \sum_{k=1}^{\mathrm{MAX}} B(k) \sin \pi kx/L,$$

i.e., the edge function with the linear trend removed.[6]

Rewritten to emphasize the discreteness of the data and the computational form, we have

$$H(J) = \sum_{k=1}^{\mathrm{MAX}} B(k) \sin \pi k(J - 1)/(N - 1). \tag{1}$$

The coefficients $B(k)$ are determined from

$$B(k) = [2/(N - 1)] \sum_{k=1}^{\mathrm{MAX}} H(J) \cdot \sin \pi k(J - 1)/(N - 1).$$

The mechanics of the fitting operation are discussed in detail in Lanczos[6] and, hence, will not be repeated here. The essential conditions amount to ensuring that the sum-orthogonality conditions always prevail, namely

$$\sum_{J=1}^{N} \sin [\mathrm{ARG}\ (J)] = 0$$

$$\sum_{J=1}^{N} \sin^2 [\mathrm{ARG}\ (J)] = \frac{N-1}{2}.$$

Rigorous imposition of these conditions implies the constraint $H(1) = H(N)$.

The critical problem is the determination of the parameter MAX in Eq. (1) in order to secure the best representation of the signal in the presence of noisy data. (If the data is not noisy, the metrologist is immensely lucky and cannot count on it happening again, or the measurement process is inefficient). If MAX is too small, information is lost, and data recovered from $H(x)$, regarded as an interpolation function, is unduly smoothed. If MAX is too large, the error (e.g; equivocation and noise) will be too heavily included in the approximating function. This may have disastrous results in the application of $H(x)$ to certain practical problems. If MAX $= N - 1$, the greatest value it can have, the approximating function will pass through every measured point identically (2 points per shortest wave) and hence it will include all of both signal and noise. From the physical point of view, this approximation, which is purely geometrical, is dangerous because no degrees of freedom are left for estimation of the error. There are problems where $H(x)$ with MAX $= N - 1$ can be the whole of the legitimate experience. In fact, it can be argued that all problems of measurement are of this type. In these cases no alternate source of estimate of error is possible outside of the measurements which determine $H(x)$. The critical problem, then, in measuring the optical transfer properties of lenses is that of cutting off the approximating series at MAX. This problem is not unique to the edge function method, but is common to approximation functions in general.

When MAX $= N - 1$, $H(x)$ is identical with the observations, i.e., $H(x) = H(J)$ observed. When the series is terminated at MAX $= N - 1 - M$, M being the number of terms in (1) rejected by truncation,

$$H(J)_{\mathrm{calc}} = \sum_{k=1}^{N-1-M} B(k) \sin \pi k(J - 1)/(N - 1); \quad (2)$$

$$\sum_{J=1}^{N} \frac{(H_{\mathrm{obs}} - H_{\mathrm{calc}})^2}{N - 1} = \tfrac{1}{2} \sum\nolimits_{\mathrm{Reject}} B(k)^2 = S^2. \quad (3)$$

Half the sum of the squares of the rejected terms is the mean squared residue between the observed and the calculated values. This is the residue which is minimized when the $B(k)$ are computed directly by the method of least squares. The orthogonality conditions lead to alternative procedures for computing S^2 when the data points appear only as components of partial sums, where the approximating series is computed on-line and the original data are not retained in memory.

If the range, L, is properly chosen to embrace the whole of the edge function from zero slope to zero slope, and if the machine is designed and operated in a manner which randomizes the error,[7] and if the approximating function is truncated so that $(H_{\mathrm{obs}} - H_{\mathrm{calc}})$ is essentially all noise; S^2 is an estimate of the variance of error, and M less corrections for additional constraints is the number of degrees of freedom (d.f.) available for its estimate. The number of d.f. can range from zero upwards, a small number, and hence the probability associated with the error is evaluated through the "Student" cumulative distribution function,[8] rather than the Normal distribution. A reliability specification clarified as an error rate relative to a null hypothesis regarding deviation from the approximating function defines the "Student" T in a given application. Thus, $\mu = T\sqrt{S^2/M}$ is the range of error + and − about a zero mean estimated from S^2 on the basis of M d.f., when the probability of deviations outside this range is determined by T.

These considerations lead, then, to the following procedure for truncating the series (1). For each value of $M = (N - 1) - k - 1$ (number of data intervals—number of coefficients used−1 constraint) the computer calculates the value $T(M)\sqrt{S^2/M}$ and finds the value k corresponding to the minimum in this function. $k =$ MAX is then the number of coefficients representing $H(x)$ with minimum reliable error. The values of $T(M)$ are taken from a standard table corresponding to the desired reliability specification, and the minimum found from the application of this technique is a measure of the random contributions from the metrological system. A sudden change in this quantity is cause for inspection of the machine.

The two principal sources of error which must be randomized are the photon-arrival noise and the seismic noise due to vibrations in the building. These are controlled by the photomultiplier load and machine sample time, as noted above. All other known spurious signals have been carefully reduced to negligible proportions by such techniques as adequate grounding and bonding, light leak elimination, etc. The result of the fitting and truncation operations is an interpolation function which fits the data within a minimum reliable error as determined by the measurements themselves.

All remaining operations are purely mathematical manipulations. The series is differentiated with or without sigma smoothing[6] as desired, to yield an interpolation function for $L(x)$. This in turn may be evaluated in −pi to +pi corresponding to the slit displacement, tabulated if desired, and the data transformed on any arbitrary frequency intervals to yield the OTF. At this point, provided the frequencies of interest are not too near the zeros of the object slit, the contributions of the slit may be removed and the result inverted to yield the true $L(x)$

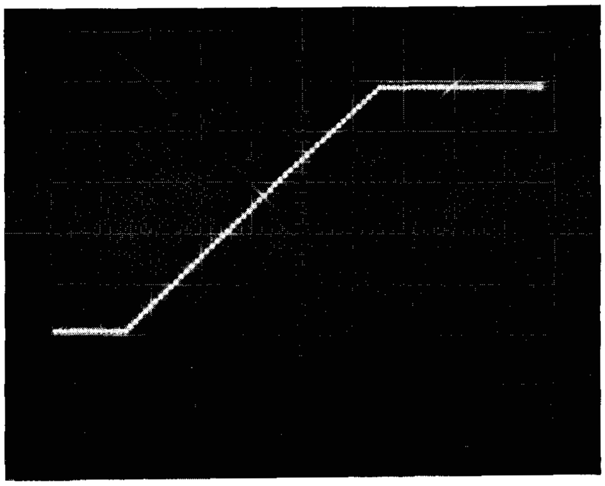


Figure 8 Simulated edge function—a ramp.

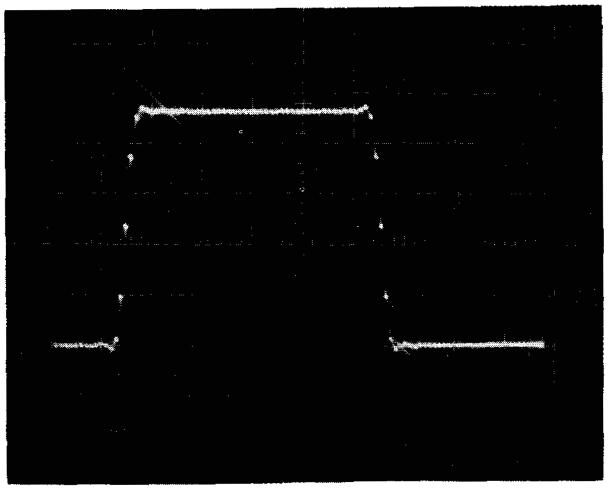


Figure 9 Line spread function. Derivative of waveform in Fig. 8.

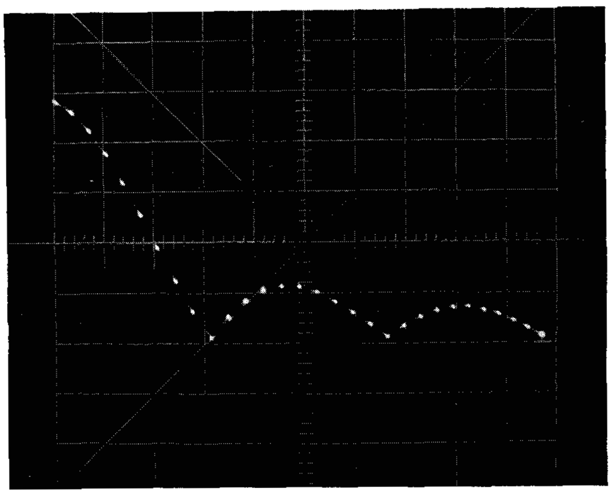


Figure 10 Modulation transfer function: Fourier transform of line spread function in Fig. 9.

Table 1 Comparison of calculated values of Fourier transform of pulse with independent calculation of sin $(x)/x$.

Cycles/millimeter	*Transform*	*Sin (x)/x*
1.70	0.9835	0.9836
3.40	0.9351	0.9354
5.10	0.8576	0.8583
6.80	0.7557	0.7568
8.50	0.6352	0.6366
10.20	0.5029	0.5045
11.90	0.3663	0.3678
13.60	0.2325	0.2338
15.30	0.1085	0.1092
17.00	0.0000	0.0000
18.70	0.0884	0.0894
20.40	0.1539	0.1559
22.10	0.1951	0.1980
23.80	0.2125	0.2162
25.50	0.2080	0.2122
27.20	0.1849	0.1892
28.90	0.1476	0.1514
30.60	0.1010	0.1039
32.30	0.0501	0.0517
34.00	0.0000	0.0000
35.70	0.0450	0.0468
37.40	0.0814	0.0850
39.10	0.1068	0.1119
40.80	0.1198	0.1261
42.51	0.1204	0.1273
44.21	0.1096	0.1164
45.91	0.0893	0.0953
47.61	0.0623	0.0668
49.31	0.0314	0.0339
51.01	0.0000	0.0000

Table 2(a) Measurement data: 10×, 0.25N.A.microscope objective tested at 20×, Wratten No. 55 filter. (See text for explanation of data in this and subsequent tables.)

0.25 NA 47 pts 1
Gmag = 20.00 XLU = 9.10 Microns, N = 47
17 Error = 0.759
Shift = 11.27 degrees

Frequency	*Amplitude*	*Phase*
35.00	.97691	0.000
70.00	.91549	.476
105.00	.83424	1.615
140.00	.75108	3.134
175.00	.67477	4.476
210.00	.60535	5.318
245.00	.54034	5.860
280.00	.47893	6.554
315.00	.42133	7.613
350.00	.36663	8.828
385.00	.31293	9.880
420.00	.26011	10.751
455.00	.21136	11.711
490.00	.17097	12.847
525.00	.14037	13.648
560.00	.11679	13.517
595.00	.09592	12.824
630.00	.07535	13.018
665.00	.05576	15.685
700.00	.03922	20.947

due to the lens alone. The object slit in the subject machine has a width of 10.5 microns, giving a first zero in the neighborhood of 95 cycles/millimeter at 1 to 1 magnification. Since in most cases the contributions are negligible, the operator has the option of eliminating this final step.

The display of the minimum reliable error and the ability to adjust the number of recorded data points provides the operator with a means of altering the system throughput speed to meet accuracy requirements which may be less than the best that the machine can deliver.

Typical results

To investigate the validity of the mathematics in the program, simulated edge function and appropriate calibration data in the form of a ramp were punched in cards. The line spread function resulting from this simulated edge function should be a pulse, and the Fourier transform should be sin $(x)/x$. The graphs in Figs. 8 and 9 show the edge function and the resultant line spread function. The sine series contained 50 terms. The steep rise of the pulse and the very small overshoot are evident. The width of the pulse at the half-amplitude points was determined from a tabulation of the line spread function. This width of 0.05881 millimeters corresponds to a frequency of 17.004 cycles/millimeter. This frequency should have zero amplitude in the Fourier transform, as should the second and third harmonics. The tabulated Fourier transform, and an independent calculation of sin $(x)/x$ are given in Table 1 for comparison purposes. A graph of the transform is shown in Fig. 10.

A test of the stability of the machine and the validity of the focal adjustment procedure was made using the microscope objective lens taken from a commerical microdensitometer. The test was conducted at a nominal magnification of 20×, which is the normal operating condition for the microdensitometer. The tabulated data in Tables 2(a), (b) and (c) were repeated measurements with no mechanical changes other than thermal drifts. The data in Tables 3(a), (b) and (c) were taken by moving the lens out of focus and then refocussing each time.

Each of the tabulations in Tables 2 and 3 were taken from the console typewriter. The first line gives the identifying information for the particular measurement. The second line gives the system magnification, the length of the sweep in the image plane, and the number of data points taken. The third line shows the number of terms in the approximating function and the error, in parts per thousand, between the series and the parent data. The fourth line is the amount of decentration between the maximum slope of the edge function and the center of the measurement interval, expressed in degrees of the first OTF frequency. This is used as a normalizing factor to remove the linear trend in the phase angle. The three columns of data below the fourth line are the frequencies in cycles/millimeter, the amplitude, and the phase angle in degrees of the OTF.

Table 2(b) Repeat of Table 2(a). No mechanical changes.

0.25 NA 47 pts 2
Gmag = 20.00 XLU = 9.10 microns, N = 47
22 Error = 0.885
Shift = 12.60 degrees

Frequency	*Amplitude*	*Phase*
35.00	0.97767	0.000
70.00	0.91798	.448
105.00	0.83804	1.519
140.00	0.75459	2.974
175.00	0.67633	4.341
210.00	0.60444	5.352
245.00	0.53789	6.154
280.00	0.47646	7.051
315.00	0.41978	8.102
350.00	0.36574	9.076
385.00	0.31189	9.806
420.00	0.25857	10.469
455.00	0.20955	11.412
490.00	0.16902	12.638
525.00	0.13774	13.570
560.00	0.11274	13.823
595.00	0.09046	14.195
630.00	0.06968	16.446
665.00	0.05147	21.782
700.00	0.03653	28.754

Table 2(c) Repeat of Table 2(a). No mechanical changes.

0.25 NA 47 pts 3
Gmag = 20.00 XLU = 9.10 microns, N = 47
15 Error = 0.641
Shift = 13.02 degrees

Frequency	*Amplitude*	*Phase*
35.00	0.97806	0.000
70.00	0.91853	.389
105.00	0.83699	1.385
140.00	0.75051	2.881
175.00	0.67018	4.448
210.00	0.59863	5.629
245.00	0.53339	6.294
280.00	0.47163	6.724
315.00	0.41247	7.320
350.00	0.35643	8.268
385.00	0.30393	9.404
420.00	0.25491	10.411
455.00	0.20972	11.142
490.00	0.16977	11.750
525.00	0.13665	12.539
560.00	0.11054	13.705
595.00	0.08961	15.392
630.00	0.07134	18.152
665.00	0.05441	23.334
700.00	0.03958	32.826

Table 3(a) Measurement data: 10×, 0.25 N.A. microscope objective at 20× Wratten No. 55 filter, refocused.

Refocused 1
Gmag = 20.00 XLU = 9.10 microns, N = 47
25 Error = 0.909
Shift = 16.11 degrees

Frequency	*Amplitude*	*Phase*
35.00	0.97986	0.000
70.00	0.92490	.300
105.00	0.84869	1.060
140.00	0.76644	2.174
175.00	0.68843	3.300
210.00	0.61774	4.113
245.00	0.55315	4.598
280.00	0.49313	5.032
315.00	0.43715	5.635
350.00	0.38429	6.277
385.00	0.33228	6.604
420.00	0.27962	6.486
455.00	0.22833	6.215
490.00	0.18346	6.126
525.00	0.14871	5.839
560.00	0.12263	4.236
595.00	0.09993	.914
630.00	0.07635	−2.761
665.00	0.05195	−4.007
700.00	0.03066	.592

Table 3(b) Same as Table 3(a) but refocused again using derivative.

Refocused 2
Gmag = 20.00 XLU = 9.10 microns, N = 47
13 Error = 0.683
Shift = 14.48 degrees

Frequency	*Amplitude*	*Phase*
35.00	0.97750	0.000
70.00	0.91650	.364
105.00	0.83307	1.277
140.00	0.74458	2.590
175.00	0.66196	3.865
210.00	0.58773	4.170
245.00	0.51996	5.164
280.00	0.45725	5.658
315.00	0.40005	6.563
350.00	0.34843	7.724
385.00	0.30011	8.560
420.00	0.25211	8.702
455.00	0.20455	8.473
490.00	0.16161	8.688
525.00	0.12803	9.682
560.00	0.10426	10.270
595.00	0.08550	8.693
630.00	0.06629	4.707
665.00	0.04503	−.377
700.00	0.02445	−4.741

Table 3(c) Same as Table 3(a) but refocused again using derivative.

Refocused 3
Gmag = 20.00 XLU = 9.10 microns, N = 47
14 Error = 0.784
Shift = 15.53 degrees

Frequency	*Amplitude*	*Phase*
35.00	0.97665	0.000
70.00	0.91595	.707
105.00	0.83862	2.319
140.00	0.76136	4.316
175.00	0.68808	5.944
210.00	0.61588	7.018
245.00	0.54519	8.031
280.00	0.48113	9.466
315.00	0.42682	11.126
350.00	0.37858	12.376
385.00	0.33002	13.070
420.00	0.27959	13.857
455.00	0.23208	15.478
490.00	0.19304	17.708
525.00	0.16269	19.240
560.00	0.13701	19.255
595.00	0.11307	18.514
630.00	0.09111	18.387
665.00	0.07210	19.016
700.00	0.05501	18.713

Examination of the last three sets of data shows that the focal setting repeats to a high degree. The interpretation of best focus is usually a matter of taste and requires several measurements at slightly differing focal settings. The derivative focusing method has been found to yield the highest overall values for modulation in a large sample of production lenses.

Conclusion

The present measuring system is an outgrowth of a need to measure the modulation transfer properties of microscope objectives of the highest quality. Due to the generally accepted practical limit of 200 cycles/millimeter for systems using moving targets, and the inflexibility in wavelength afforded by the interferometric methods, these measurement philosophies were ruled out from the very beginning. The expected practical limit of resolution available with the edge function analysis approach has not been demanded by any of the lenses thus far examined, including the best commercially available microscope objectives.

The employment of a high-speed digital computer as the logical connecting element between several well-known instrumental and mathematical techniques has produced a demonstrably stable, reproducible, and flexible tool.

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Received August 20, 1968